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The Existence of Periodic Orbits on the Sphere

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Introduction

In the theory of dynamical systems, there remains the open problem, so called Seifert Conjecture: Has any sufficiently smooth flow on S^3 a periodic orbit? This conjecture is based on Seifert's paper [11] which proved the following theorem.

THEOREM 1. *Let $x=(x_1, x_2)$, $y=(y_1, y_2)$ be points of $\boldsymbol{R}^2$ and consider the following equation in $\boldsymbol{R}^4$*

$$\dot{x}_i=y_i,\ \dot{y}_i=-x_i;\ i=1, 2 . \tag{1}$$

This system has $S^3=\{(x, y)\in \boldsymbol{R}^4; x_1^2+x_2^2+y_1^2+y_2^2=1\}$ as an invariant set, so we can consider the flow on S^3 induced by (1). Then any flow C^0 near the above flow on S^3 has at least one periodic orbit.

The system (1) is the Hamiltonian system with Hamiltonian

$$H(x, y)=\frac{1}{2}(y_1^2+y_2^2)+\frac{1}{2}(x_1^2+x_2^2) , \tag{2}$$

which describes the harmonic oscilaters.

More strongly, (2) is derived from the Lagrangian system

$$\frac{d}{dt}\frac{\partial}{\partial \dot{x}_i}(T-U)=\frac{\partial}{\partial x_i}(T-U);\ i=1, 2 \tag{3}$$

where

$$T=\frac{1}{2}(\dot{x}_1^2+\dot{x}_2^2) \quad \text{and} \quad U=\frac{1}{2}(x_1^2+x_2^2) , \tag{4}$$

with $y_i=(\partial T/\partial \dot{x}_i)=\dot{x}_i$ $(i=1, 2)$.

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