TOKYO J. MATH.
VOL. 8, NO. 1, 1985

A Remark on the Duality Mapping on l^∞

Marián FABIAN

Prague

(Communicated by J. Wada)

Here we answer a question raised in [1]. In order to formulate this question we are to recall some notations and facts from [1]. We work with the dual $l^{\infty*}$ of l^∞, which can be written as the direct sum $l^1+c_0^\perp$. S denotes the unit sphere in l^∞ and $\mathrm{sm}\,S$ the set of the smooth points of S. The duality mapping F_0: $S\to 2^{l^{\infty*}}$ is defined as follows

$$F_0(v)=\{\lambda\in l^{\infty*}: \lambda(u)=1=\|\lambda\|\}\ , \qquad v\in S\ .$$

$\mathrm{ext}\,F_0(v)$ denotes the set of extremal points of $F_0(v)$. The mentioned question sounds as:

"*Given* $v\in S\backslash \mathrm{sm}\,S$ *and* $\lambda\in \mathrm{ext}\,F_0(v)$, *does there exist a sequence* $\{v_n\}\subset \mathrm{sm}\,S$ *such that* $\|v_n-v\|\to 0$ *and that* λ *is a* w^**-cluster point of the sequence* $\{F_0(v_n)\}$?"

The answer is negative in general as it follows from Propositions 1 and 2. Owing to some reasons from [1] we may and do restrict ourselves to the situation when $v\geqq 0$ and $\lambda\in c_0^\perp$.

We recall that (see [1]) there is a one-to-one correspondence between *ultrafilters* and 0-1-*measures*, namely, given an ultrafilter $\mathscr{U}$ on the set of natural numbers N we can define the measure on N as

$$(*)\qquad \lambda(A)=\begin{cases}1 & \text{iff} \quad A\in\mathscr{U}\\ 0 & \phantom{\text{iff}} \quad A\notin\mathscr{U}\end{cases}$$

and conversely. Also, a 0-1-measure λ is in $c_0^\perp$ if and only if the corresponding $\mathscr{U}$ is free (non-principal), i.e., $\mathscr{U}$ contains no finite sets. It is known [1] that, for $v\in S$, $v\geqq 0$, $\mathrm{ext}\,F_0(v)$ consists only of 0-1-measures.

PROPOSITION 1. *Let* $v\in S\backslash\mathrm{sm}\,S$, $v\geqq 0$, $\lambda\in \mathrm{ext}\,F_0(v)\cap c_0^\perp$ *and* $\mathscr{U}$ *be the ultrafilter associated with* λ *by* $(*)$. *Then the following assertions are equivalent*:

Receved December 10, 1983