TOKYO J. MATH.
VOL. 9, No. 1, 1986

On the Convolution of Functions of Two Variables and Generalized Harmonic Analysis

Dedicated to Professor Hisaharu Umegaki on his sixtieth birthday

Katsuo MATSUOKA

Takamatsu National College of Technology

(Communicated by T. Saito)

Introduction

In the linear filter theory, Wiener considered especially a weighting K in the time domain, i.e. the filters $K*$ for which the response g to an input signal f is given by

$$g(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} K(t-\tau)f(\tau)d\tau=(K*f)(t)\ , \quad t\in(-\infty,\ \infty)\ .$$

Also, he indicated the importance of admitting as inputs arbitrary signals of the class S. His main theorem in [9] is: If

$$(1+|t|)K(t)\in L^1\cap L^2(-\infty,\ \infty)\ ,$$

then the responce of the filter K to a signal f in S is a signal $g\in S'$, by using the generalized harmonic analysis (cf. Masani [5]).

In this paper, we shall extend this result to the case of functions of two variables under a restricted rectangular mean concerning the double limit process, using the generalized harmonic analysis of functions of two variables in Matsuoka [6].

Wiener has proved a Tauberian theorem in a generalized sense, with respect to a weighted moving average of a function which is bounded on the average. On the other hand, in Anzai, Koizumi and Matsuoka [1], we have considered the form of general Tauberian theorems about a weighted moving average $K*f$ of f. We shall also extend the above theorem of Wiener to the case of functions of two variables under a restricted rectangular mean concerning the double limit process in consideration of the modified form.

Received December 19, 1984