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Minimal Tori in S^3 Whose Lines of Curvature Lie in S^2

Dedicated to Professor Morio Obata on his 60th birthday

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Introduction

Let $\varphi: \Sigma \rightarrow S^3$ be a minimal immersion of a compact orientable surface Σ into the unit 3-sphere S^3. It is valuable to study the set of such immersions with Σ of given genus. For example, when Σ is of genus 0, i.e., Σ is the 2-sphere, φ must be the totally geodesic immersion of S^2 into S^3 [3] [1] [4].

Assume Σ is the torus. In this case, there is the well-known minimal isometric *embedding* of the flat square torus $S^1(1/\sqrt{2}) \times S^1(1/\sqrt{2})$ into S^3 called *the Clifford immersion*. Though there are many minimal immersions of the torus into S^3, they are not embedded. Thus, it is conjectured that the only minimal embedding of the torus into S^3 is the Clifford one [7].

To study this, we consider minimal immersions of a torus into S^3 having the following property:

(*) Each line of curvature of the immersions lies in some totally geodesic 2-sphere in S^3.

The main theorem of this paper is the following:

THEOREM. (1) *There exist infinitely many minimal immersions of the torus into S^3 satisfying* (*).

(2) *A minimal immersion of the torus into S^3 satisfying* (*) *is not an embedding provided that it is congruent with the Clifford one.*

§1. Preliminaries.

Let $\varphi: \Sigma \rightarrow S^3$ be a smooth immersion of a surface into the unit 3-

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