Kaehler Submanifolds of Complex Space Forms

Dedicated to Professor Morio Obata on his 60th birthday

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Introduction

E. Calabi [1] proved that a complex linear space, a complex hyperbolic space and a complex projective space can not be holomorphically and isometrically immersed in each other. In this paper we show that Kaehler submanifolds of complex space forms of different types are essentially different from each other. Namely we prove the following:

THEOREM. *Any two of complex space forms of different types have no Kaehler submanifold in common, that is,*

(1) *A Kaehler submanifold of C^N can not be a Kaehler submanifold of any complex hyperbolic space.*

(2) *A Kaehler submanifold of C^N can not be a Kaehler submanifold of any complex projective space.*

(3) *A Kaehler submanifold of a complex hyperbolic space can not be a Kaehler submanifold of any complex projective space.*

It should be remarked that no global assumption is made in the theorem, namely it is local in nature. In the proof of the theorem, the notion "diastasis" introduced by E. Calabi [1] plays an essential rôle. Though the diastasis depends only on the metric, it is compatible with that of an ambient space. Using this property of diastasis, a necessary and sufficient condition for a Kaehler manifold to be holomorphically and isometrically immersed into a complex space form has been obtained by E. Calabi [1]. Then the proof of our theorem reduces to the local rigidity theorem for isometric mappings of Kaehler submanifolds of C^N into the Hilbert space l^2.

The idea of the diastasis here can be applied to a wider class of