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A Note on the Transcendental Continued Fractions

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Introduction

Let $a_1, a_2, a_3, \cdots; b_1, b_2, b_3, \cdots$ be all integers and $a_1 \geqq 0$, $a_2 > 0$, $a_3 > 0, \cdots$; $b_1 \geqq 0$, $b_2 > 0$, $b_3 > 0$, $\cdots$ all along this note.

In [2], G. Nettler proved the following theorem.

NETTLER'S THEOREM. *For*

$$A = a_1 + \frac{1}{a_2 +}\frac{1}{a_3 +}\cdots \quad \textit{and} \quad B = b_1 + \frac{1}{b_2 +}\frac{1}{b_3 +}\cdots,$$

if $a_n > b_n > a_{n-1}^{(n-1)^2}$ for all n sufficiently large, then A, B, $A \pm B$, A/B and AB are all transcendental numbers.

The aim of this note is to prove the following theorem that is an improvement of Nettler's Theorem.

THEOREM. *For*

$$A = a_1 + \frac{1}{a_2 +}\frac{1}{a_3 +}\cdots \quad \textit{and} \quad B = b_1 + \frac{1}{b_2 +}\frac{1}{b_3 +}\cdots,$$

if $a_n > b_n > a_{n-1}^{\gamma(n-1)}$ for all n sufficiently large, then, A, B, $A \pm B$, A/B and AB are all transcendental numbers, where γ is any constant such that $\gamma > 16$.

We give an elementary proof of this theorem using the method of G. Nettler.

§1. Lemmas.

LEMMA 1. *If*

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