TOKYO J. MATH.
VOL. 10, NO. 1, 1987

# On the Number of Parameters of Linear Differential Equations with Regular Singularities on a Compact Riemann Surface

Dedicated to Professor Kôtaro Oikawa on his 60th birthday

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(Communicated by R. Takahashi)

## Introduction

Let $X$ be a compact Riemann surface of genus $g$ and let $Y$ be a divisor of $X$ consisting of $m$ distinct points $p_1, \cdots, p_m$ of $X$. *We suppose that $m \geqq 1$ and moreover $m \geqq 2$ when $g=0$.* We recall a fundamental fact about linear differential equations with regular singularities; let $\Delta = \{z \in \boldsymbol{C} \mid |z|<1\}$ be a unit disc in $\boldsymbol{C}$ and let

$$(1) \qquad \frac{d^n w}{dz^n} + a_1(z)\frac{d^{n-1}w}{dz^{n-1}} + \cdots + a_n(z)w = 0$$

be a linear differential equation of order $n$ where $a_i(z)$ is holomorphic in $\Delta - \{0\}$. The origin 0 is said to be a *regular singular point* of the equation (1) if the functions $z^i a_i(z)$ $(i=1, 2, \cdots, n)$ are holomorphic at 0. It is well known that this is equivalent to the condition that the equation (1), multiplied by $z^n$, can be written in the form

$$(2) \qquad \left(z\frac{d}{dz}\right)^n w + b_1(z)\left(z\frac{d}{dz}\right)^{n-1} w + \cdots + b_n(z)w = 0$$

where $b_i(z)$ $(i=1, \cdots, n)$ are holomorphic at 0. Using this fact, we define a linear differential equation on a compact Riemann surface $X$ of order $n$ with regular singularities along $Y$ as follows; let $X = \cup_{j=1}^{N} U_j$ be a sufficiently fine finite open coordinate covering of $X$ such that $p_j \in U_j$ $(j=1, \cdots, m)$ and $z_j(p_j)=0$ for $j=1, \cdots, m$ and $z_j$ is *nowhere zero* in $U_j$ for $j=m+1, \cdots, N$. In each neighbourhood $U_j$ we consider a linear differential equation

$$(3) \qquad \left(z_j\frac{d}{dz_j}\right)^n w + b_{j,1}(z_j)\left(z_j\frac{d}{dz_j}\right)^{n-1} w + \cdots + b_{j,n}(z_j)w = 0$$

Received July 1, 1986