TOKYO J. MATH.
VOL. 10, NO. 2, 1987

On the Symmetry of a Reflecting Brownian Motion Defined by Skorohod's Equation for a Multi-Dimensional Domain

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Introduction

The existence and uniqueness of solutions to Skorohod's equation for a multi-dimensional domain were discussed by Lions and Sznitman [6] and Saisho [9]. In this paper we prove that a reflecting Brownian motion X obtained by solving Skorohod's equation for a domain D in $\boldsymbol{R}^d$ is symmetric in the sense that $\int fT_t g dx = \int gT_t f dx$ holds for any L^2-functions f and g on $\bar{D}$, where T_t is the semigroup of X. The proof is based on the construction of X by the penalty method, that is, we prove the above result by showing that X can be approximated by symmetric diffusions (with respect to the invariant measures) which are described by stochastic differential equations with smooth drift coefficients of gradient type. The penalty method was used for the study of reflecting diffusions by Lions, Menaldi and Sznitman [5], Menaldi [7] and Menaldi and Robin [8]. Our method is similar to theirs but our approximation result (Theorem 2) is given in a pathwise formulation and improves some results of [6].

§1. Formulation of the problem and the result.

We denote by $B(x, r)$ the open ball in $\boldsymbol{R}^d$ with center x and radius r and write $\langle \cdot, \cdot \rangle$ for the usual inner product in $\boldsymbol{R}^d$. Let D be a domain in $\boldsymbol{R}^d$ and let $x \in \partial D$. Denote by $\mathfrak{N}_{x,r}$ the set of unit vectors $\boldsymbol{n}$ in $\boldsymbol{R}^d$ such that $B(x-r\boldsymbol{n}, r) \cap D = \emptyset$ and by $\mathfrak{N}_x$ the union of $\mathfrak{N}_{x,r}$ as r runs over all positive numbers. An element of $\mathfrak{N}_x$ is called an inward normal vector at x.

Following Lions and Sznitman [6] we introduce conditions for D.

CONDITION (A). There exists a constant $r_0 > 0$ such that $\mathfrak{N}_x = \mathfrak{N}_{x,r_0} \neq \emptyset$ for any $x \in \partial D$.

Received October 27, 1986