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On Some Integral Invariants, Lefschetz Numbers and Induction Maps

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Dedicated to Professor Hiroshi Toda on his 60th birthday

§1. Introduction.

Let M be a compact complex manifold, G any compact subgroup of the complex Lie group of all holomorphic automorphisms of M and $\mathfrak{G}$ the Lie algebra of G which consists of holomorphic vector fields on M. In [5], the first author defined a character $f: \mathfrak{G} \to \boldsymbol{C}$ (more generally defined a $\boldsymbol{C}$-character of the complex Lie algebra of all holomorphic vector fields on M) which depends only on the complex structure of M and vanishes if M admits a Kaehler-Einstein metric. In this paper, we first see that characters of this kind appear naturally in the Lefschetz numbers. More precisely, let $\mathscr{D}$ be the Dolbeault complex of M with values in a certain holomorphic vector bundle over M and H^i the i-th cohomology group of $\mathscr{D}$. Then the Lefschetz number $L(g)$, for $g \in G$, is by definition

$$L(g)=\sum_i (-1)^i \operatorname{tr}(g|_{H^i}) .$$

In Theorem 4.3, we show that $f(X)$, for $X \in \mathfrak{G}$, coincides up to constant with *the second term of the Taylor expansion of* $L(\exp tX)$ whose first term is of course the arithmetic genus of $\mathscr{D}$. Then it becomes clear that f depends only on the complex structure of M and that $f(\mathrm{Ad}(g)X)=f(X)$ for any $g \in G$.

Now we wish to put this view point into a single diagram. Let G and H be compact Lie groups with Lie algebras $\mathfrak{G}$ and $\mathfrak{H}$. Let M be a compact oriented manifold of dimension $2m$ and P a principal right H-bundle over M. Suppose that G acts on $P \to M$ on the left as bundle automorphisms and that the action of G on M is orientation-preserving. Let θ be a G-invariant connection of P. Then, as in [4], an H-equivariant

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