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On Two Variable p-Adic L-Functions and a p-Adic Class Number Formula

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Introduction.

Let K be an imaginary quadratic field with class number 1 and discriminant $-d_K$ lying inside the complex number field $\boldsymbol{C}$, and denote by O the ring of integers of K. Let E be an elliptic curve defined over K with complex multiplication by O. We denote by ψ the Grössencharacter of E over K, and by $\mathfrak{f}$ the conductor of ψ. Fix a Weierstrass model for E

$$y^2=4x^3-g_2x-g_3 \tag{0.1}$$

such that g_2, $g_3\in O$ and the discriminant $\Delta=g_2^3-27g_3^2$ of (0.1) is divisible only by primes dividing $6\mathfrak{f}$. Let $P(z)$ be the Weierstrass pe-function associated with (0.1), and L the period lattice of $P(z)$. Fix an element $\Omega_\infty\in L$ such that $L=\Omega_\infty O$.

Let p be a rational prime number prime to $6d_K\mathfrak{f}$ and we assume that p splits in K, say $(p)=\mathfrak{p}\bar{\mathfrak{p}}$. We denote by $K_\mathfrak{p}$ the completion of K at $\mathfrak{p}$ and identify $K_\mathfrak{p}$ with the rational p-adic number field $\boldsymbol{Q}_p$. Let $\boldsymbol{C}_p$ be the completion of the algebraic closure of $K_\mathfrak{p}$, and denote by I the ring of integers of $\boldsymbol{C}_p$. Let $\bar{\boldsymbol{Q}}$ denote the algebraic closure of the rational number field $\boldsymbol{Q}$ in $\boldsymbol{C}$. Fixing an embedding of $\bar{\boldsymbol{Q}}$ in $\boldsymbol{C}_p$, we regard $\bar{\boldsymbol{Q}}$ also as a field contained in $\boldsymbol{C}_p$.

If Ψ is a Grössencharacter of K, we denote by $L(\Psi, s)$ the primitive complex Hecke L-function attached to Ψ. For each integral ideal $\mathfrak{a}$ of K, let $R_\mathfrak{a}$ denote the ray class field modulo $\mathfrak{a}$ of K. If $\mathfrak{a}$ is divisible by the conductor of Ψ, then, for each $\sigma\in\mathrm{Gal}(R_\mathfrak{a}/K)$, we denote by $L_\mathfrak{a}(\sigma, \Psi, s)$ the partial zeta function attached to Ψ and $\sigma\in\mathrm{Gal}(R_\mathfrak{a}/K)$.

If χ is a primitive class character of K, we put

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