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Some Results on Additive Number Theory V

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§1. The main theorem.

Let $\omega(n)$ denote the number of distinct prime factors of a positive integer n.

Let k and l be positive integers;
let $a_1, \cdots, a_k$ be distinct *non-zero* integers;
let $a_{k+1}, \cdots, a_{k+l}$ be distinct integers.

We put, for $\alpha_i < \beta_i$, $i=1, \cdots, k+l$,

$$\Phi(\alpha_i, \beta_i) = \frac{1}{\sqrt{2\pi}} \int_{\alpha_i}^{\beta_i} \exp\left(-\frac{x^2}{2}\right) dx .$$

Let N be a positive integer, which will be assumed to be sufficiently large as occasion demands.

THEOREM. *Let $A(N)=A(N; a_1, \cdots, a_{k+l}; \alpha_1, \beta_1, \cdots, \alpha_{k+l}, \beta_{k+l})$ denote the number of representations of N as the sum of the form $N=p+n$, where p is prime, and n is a positive integer such that*

$$\log\log N + \alpha_i \sqrt{\log\log N} < \omega(p+a_i) < \log\log N + \beta_i \sqrt{\log\log N}$$

for $i=1, \cdots, k$, and

$$\log\log N + \alpha_i \sqrt{\log\log N} < \omega(n+a_i) < \log\log N + \beta_i \sqrt{\log\log N}$$

for $i=k+1, \cdots, k+l$ simultaneously. Then, as $N \to \infty$, we have

$$A(N) \sim \frac{N}{\log N} \cdot \prod_{i=1}^{k+l} \Phi(\alpha_i, \beta_i) .$$

The paper will be read without making any references to author's previous papers, except for the proof of Lemma 4.

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