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On the Triviality Index of Knots

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§1. Introduction.

In [6] the first author derived a new numerical invariant, denoted by $O(K)$, of knots from their diagrams and showed that if the Conway polynomial of a knot K is not one, then $O(K)$ is finite ([6] Corollary 2.4). In this paper, we call $O(K)$ the triviality index of K. It arises a problem as to whether or not there exists a knot K such that $O(K)=n$ for any natural number n.

In this paper, we show the following theorems.

THEOREM A. *If a knot K has a $2n$-trivial diagram $(n>1)$, the coefficient of z^{2n} of the Conway polynomial of K is even.*

THEOREM B. *For any natural number n with $n>1$, there exist infinitely many knots K's with $O(K)=n$.*

Moreover in the case $O(K)=3$ we show the following.

THEOREM C. *Let $f(z)=1+\sum_{i=2}^{l} a_{2i}z^{2i}$, where a_{2i} $(2\leqq i\leqq l)$ are integers. If a_4 is odd, there is a knot K such that $O(K)=3$ and the Conway polynomial of K is $f(z)$.*

Throughout this paper, we work in PL-category and refer to Burde and Zieschang [1] and Rolfsen [8] for the standard definitions and results of knots and links.

§2. Definitions and facts.

The Conway polynomial $\nabla_L(z)$ ([2]) and the Jones polynomial $V_L(t)$ ([3]) are invariants of the isotopy type of an oriented knot or link in a 3-sphere S^3. The Conway polynomial is defined by the following formulas:

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