

Homogeneous Harmonic Maps into Complex Projective Spaces

Yoshihiro OHNITA

Tokyo Metropolitan University

Dedicated to Professor Tadashi Nagano on his sixtieth birthday

Introduction.

Let $M=G/K$ be a compact homogeneous space of a compact Lie group G with a G -invariant Riemannian metric g and N be a Riemannian manifold. A homogeneous harmonic map from M to N means a ρ -equivariant harmonic map from M to N relative to a homomorphism ρ of G to the isometry group of N . The existence and construction of harmonic maps are interesting and important problems in various situations. Homogeneous harmonic maps make a simple and nice class of harmonic maps. Concerning the existence of homogeneous harmonic maps, in general it is known by the idea of W.-Y. Hsiang that given a non-constant ρ -equivariant map φ from a compact homogeneous Riemannian manifold M to a compact Riemannian manifold N , then φ can be deformed to a nonconstant ρ -equivariant harmonic map φ_1 through a smooth homotopy of ρ -equivariant maps $\varphi_t (t \in [0, 1])$ with $\varphi_0 = \varphi$ (cf. [Gu 5]). Naturally we are interested in getting more explicit descriptions and detailed properties of homogeneous harmonic maps for specific homogeneous Riemannian manifolds M and N . In his nice paper [Gu 1], Guest pointed out many interesting connections of the research for homogeneous harmonic maps with problems of differential geometry and mathematical physics. Moreover he gave algebraic descriptions of the harmonic map equation for general homogeneous maps, and discussed the harmonicity of homogeneous maps into complex projective spaces and a construction of homogeneous harmonic maps from flag manifolds into complex Grassmann manifolds by the method of osculating flags and twistor spaces (cf. [E-W]). In this paper we discuss homogeneous harmonic maps and minimal immersions into complex projective spaces in detail, by using

Received May 15, 1989

Revised July 15, 1989