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Stable Sheaves of Rank 2 on a 3-Dimensional Rational Scroll

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§0. Introduction.

In a joint work ([HI]), we showed that a family of stable vector bundles of rank 2 on a 3-dimensional rational scroll forms a complement of a dual 3-dimensional rational scroll. It is natural to ask what is its closure in the moduli of stable sheaves of same rank and same Chern classes (cf [M2]). Is the moduli of stable sheaves connected? Does it have other irreducible components? We will answer such questions in this paper.

Let (X, H) be a 3-dimensional rational scroll and M be the moduli of H-stable sheaves of rank 2 on X with $C_1=C_1(\mathscr{L})$, $C_2=D\cdot F$ and $C_3=0$ (see (2.1) for notations).

In section 1, we list up several formulas and describe the normal bundle of an n-dimensional rational scroll. In section 2, we show that there are 6 types of stable sheaves. The hierarchy of such types will be settled. In section 3, we construct limits of extensions. This is the main tool of this paper. The idea comes from monad ([BH]) and elementary transformation ([M1]). The main theorem (Theorem 3.13) says that the moduli M is connected and has two irreducible components M_0 and M_1. M_0 contains all vector bundles and M_1 contains no vector bundles. The dimension of M_1 is greater than that of M_0. In section 4, we construct a family for the difference $M_0\setminus M_1$. The description of the normal bundle of a 3-dimensional rational scroll will be used to construct the family. In section 5, we construct a family for M_1. The construction will be sketched without proofs.

§1. Preliminaries.

In this section, we list up some basic formulas and describe the normal bundle of an n-dimensional rational scroll. The former will be used mainly

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