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On the Divisibility Properties of the Orders of $K_2\mathcal{O}_F$ for Certain Totally Real Abelian Fields F

Satoru SAEGUSA

Gakushuin University

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Introduction.

In [2], Hettling has proved that for any prime number q there exist infinitely many totally real abelian fields F such that q divides the orders of $K_2\mathcal{O}_F$, Milnor's K_2-groups of the rings of integers in F (cf. [4]), in discussing the divisibility properties of the orders of these groups in certain cases. In this paper, we shall show that the prime q in this proposition can be replaced by any integer $n \in N$. We shall use the same notations as in [2], as explained in the following paragraph for completeness' sake.

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§1. Notations and preliminaries.

For $m \in N$, let ζ_m be a primitive m-th root of unity, and $Q(\zeta_m)^+ := Q(\zeta_m + \zeta_m^{-1})$ the maximal totally real subfield of the full cyclotomic field $Q(\zeta_m)$. For an arbitrary abelian number field F, $\mathcal{O}_F$ denotes its ring of integers, ζ_F the Dedekind zeta-function associated to F, and H the Dirichlet character group associated to F. For a character $\chi \in H$, let $L(s, \chi)$ be the Dirichlet L-series associated to χ and $B_{i,\chi}$, $i = 1, 2, 3, \cdots$ the generalized Bernoulli numbers. The ordinary Bernoulli numbers $B_i = B_{i,1}$ belong to the principal character $\chi = 1$, refer to [7].

The Birch-Tate conjecture (cf. [1], [5]) states that

$$\sharp K_2\mathcal{O}_F = |\, W_2(F) \cdot \zeta_F(-1) \,|$$

for any totally real number field F, where

$$W_2(F) := \max\{m \in N \mid g^2 = 1 \text{ for any element } g \in \mathrm{Gal}(F(\zeta_m)/F)\} .$$

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