TOKYO J. MATH.
VOL. 14, NO. 2, 1991

Homogeneous Siegel Domains of Nonpositive Holomorphic Bisectional Curvature

Tadashi TSUJI

Mie University

(Communicated by T. Nagano)

Introduction.

The notion of quasi-symmetric Siegel domains was introduced by Satake [10] in an algebraic manner. D'Atri and Dorfmeister [2] proved that an irreducible homogeneous Siegel domain D is quasi-symmetric if and only if the Bergman metric of D induces a symmetric metric on the canonical tube domain associated with D. They proved in [3] that every quasi-symmetric Siegel domain has nonpositive holomorphic bisectional curvature relative to its Bergman metric. Besides these, few other differential geometric characterizations of quasi-symmetric Siegel domains are known. It is interesting to see whether a homogeneous Siegel domain D of nonpositive holomorphic bisectional curvature in the Bergman metric is necessarily quasi-symmetric. In this paper, we will prove it affirmatively in the following special cases:

(1) The holomorphic sectional curvature restricted on a certain submanifold of D satisfies a pinching condition;

(2) D is a domain over a dual square cone due to Xu [14];

(3) The rank of D is less than or equal to three;

(4) The dimension of D is less than or equal to ten.

We note that the same problem for the sectional curvature has been solved by D'Atri and Miatello [4].

The main results are proved in §4.

ACKNOWLEDGEMENTS. Part of this work was done while the author was visiting Rutgers University. He would like to thank the Mathematics Department of Rutgers for its hospitality. Special thanks are due to Professor Joseph E. D'Atri for various helpful discussions. Thanks are also due to the referee for suggesting several improvements of the manuscript.

Received January 18, 1991
Revised May 22, 1991