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On the Fractal Curves Induced from Endomorphisms on a Free Group of Rank 2

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0. Introduction.

Dekking showed in [3] and [4] that some endomorphisms θ on a free group of rank 2 provide us with fractal curves which induce several space tilings on $\boldsymbol{R}^2$.

In fact, let $G\langle a, b\rangle$ be a free group with generators a and b, a map $f: G\langle a, b\rangle \to \boldsymbol{Z}^2 \subset \boldsymbol{R}^2$ be a homomorphism, and L_θ be a linear representation of the endomorphism θ, that is, f and L_θ satisfies the commutative relation:

$$\begin{array}{ccc} G\langle a, b\rangle & \xrightarrow{\theta} & G\langle a, b\rangle \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle f} \\ \boldsymbol{R}^2 & \xrightarrow{L_\theta} & \boldsymbol{R}^2 . \end{array}$$

Let $K: G\langle a, b\rangle \to \boldsymbol{R}^2$ be a map which assigns to each element of $G\langle a, b\rangle$ a polygonal curve in the plane as follows: for $W = w_1 w_2 \cdots w_k \in G\langle a, b\rangle$, $K[W]$ is a polygon joining the points $f(w_1) + \cdots + f(w_j)$ $(1 \le j \le k)$ in order (exact definitions will be found in §1). In this situation, the following result is obtained.

THEOREM ([3], [4]). *Let θ be an endomorphism of $G\langle a, b\rangle$ satisfying the following conditions*:

(1) *θ has short range cancellations, that is, for any reduced word stu $(s, t, u \in \{a^{\pm 1}, b^{\pm 1}\})$, cancellation does not erase all letters of any of the subwords $\theta(s)$, $\theta(t)$ and $\theta(u)$ in $\theta(stu)$,*

(2) *L_θ is expansive, that is, the absolute values of both eigenvalues of L_θ are greater than* 1,

(3) *$K[\theta(aba^{-1}b^{-1})]$ is double point free.*

Then there exists a limit set K_θ of $L_\theta^{-n} K[\theta^n(aba^{-1}b^{-1})]$ as a curve and the set F_θ enclosed by K_θ is a space tiling set of $\boldsymbol{R}^2$:

$$\bigcup_{\alpha \in \boldsymbol{Z}^2} (F_\theta + \alpha) = \boldsymbol{R}^2 ,$$

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