Токуо Ј. Матн. Vol. 14, No. 2, 1991

On the Fractal Curves Induced from Endomorphisms on a Free Group of Rank 2

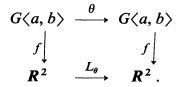
Shunji ITO and Makoto OHTSUKI

Tsuda College

0. Introduction.

Dekking showed in [3] and [4] that some endomorphisms θ on a free group of rank 2 provide us with fractal curves which induce several space tilings on \mathbb{R}^2 .

In fact, let $G\langle a, b \rangle$ be a free group with generators a and b, a map $f: G\langle a, b \rangle \rightarrow \mathbb{Z}^2 \subset \mathbb{R}^2$ be a homomorphism, and L_{θ} be a linear representation of the endomorphism θ , that is, f and L_{θ} satisfies the commutative relation:



Let $K: G\langle a, b \rangle \to \mathbb{R}^2$ be a map which assigns to each element of $G\langle a, b \rangle$ a polygonal curve in the plane as follows: for $W = w_1 w_2 \cdots w_k \in G \langle a, b \rangle$, K[W] is a polygon joining the points $f(w_1) + \cdots + f(w_j)$ $(1 \le j \le k)$ in order (exact definitions will be found in §1). In this situation, the following result is obtained.

THEOREM ([3], [4]). Let θ be an endomorphism of $G\langle a, b \rangle$ satisfying the following conditions:

(1) θ has short range cancellations, that is, for any reduced word stu $(s, t, u \in \{a^{\pm 1}, b^{\pm 1}\})$, cancellation does not erase all letters of any of the subwords $\theta(s)$, $\theta(t)$ and $\theta(u)$ in $\theta(stu)$,

(2) L_{θ} is expansive, that is, the absolute values of both eigenvalues of L_{θ} are greater than 1,

(3) $K[\theta(aba^{-1}b^{-1})]$ is double point free.

Then there exists a limit set K_{θ} of $L_{\theta}^{-n}K[\theta^n(aba^{-1}b^{-1})]$ as a curve and the set F_{θ} enclosed by K_{θ} is a space tiling set of \mathbb{R}^2 :

$$\bigcup_{\alpha\in\mathbb{Z}^2}(F_{\theta}+\alpha)=\mathbb{R}^2,$$

Received June 22, 1990