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The C^1 Uniform Pseudo-Orbit Tracing Property

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Introduction.

Let M be a closed C^∞ manifold and denote by $\mathrm{Diff}^1(M)$ the set of all diffeomorphisms of M endowed with the C^1 topology. For $\delta>0$, a sequence $\{x_i\}_{i=a}^{b-1}$ $(-\infty\le a<b\le\infty)$ is called a *δ-pseudo-orbit* for $f\in\mathrm{Diff}^1(M)$ if $d(f(x_i), x_{i+1})<\delta$ for $a\le i\le b-1$, where d is a Riemannian distance on M. Given $\varepsilon>0$, a sequence $\{x_i\}_{i=a}^{b}$ is said to be *f-ε-traced* by a point $x\in M$ if $d(f^i(x), x_i)<\varepsilon$ for $a\le i\le b$. We say that f has the *pseudo-orbit tracing property* (abbrev. POTP) if for $\varepsilon>0$ there is $\delta>0$ such that every δ-pseudo-orbit for f can be f-ε-traced by some point in M. For compact spaces these are independent of the compatible metrics used. We say that f satisfies C^1 *uniform pseudo-orbit tracing property* (abbrev. C^1-UPOTP) if there is a C^1 neighborhood $\mathfrak{U}(f)$ of f with the property that for $\varepsilon>0$ there is $\delta>0$ such that every δ-pseudo-orbit of $g\in\mathfrak{U}(f)$ is g-ε-traced by some point. By definition it is checked that C^1-UPOTP is stronger than POTP.

Robinson [5] proved that if $f\in\mathrm{Diff}^1(M)$ satisfies Axiom A and strong transversality, then f has POTP. We show the following.

THEOREM. *If $f\in\mathrm{Diff}^1(M)$ satisfies Axiom A and strong transversality, then f satisfies C^1-UPOTP.*

Combining the above theorem and a result proved in [6] we have the following corollary.

COROLLARY. *If the dimension of M is ≤ 3, then the set of all diffeomorphisms having C^1-UPOTP is characterized as the set of all Axiom A diffeomorphisms satisfying strong transversality.*

§1. Preliminary results.

Let $\Omega(f)$ be the non-wandering set of an Axiom A diffeomorphism f. The *local stable* and *unstable manifolds* are defined by

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