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Non-Existence of Homomorphisms between Quantum Groups

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§1. Introduction.

Let G be a connected complex semisimple Lie group. The (multi-parameter) quantum group $A_{\hbar,\varphi}(G)$ is a deformation of the function algebra $A(G)$ of G as a Hopf algebra (cf. [2, 5, 14, 8, 9, 10]). While the representation theory of $A_{\hbar,\varphi}(G)$ is similar to that of G, the "group theoretic" structure of $A_{\hbar,\varphi}(G)$ is rather different from that of G. For example, it seems that $A_{\hbar,\varphi}(G)$ does not have so many "subgroups" as G.

In this paper, we show that there exist no non-trivial Hopf algebra homomorphisms from $A_{\hbar,\varphi}(SL(N))$ into $A_{\hbar,\psi}(SO(N))$ $(N\geq 7)$ or $A_{\hbar,\psi}(Sp(N))$. In other words, there exists no quantum analogue of group inclusions $SO(N)\subset SL(N)$ and $Sp(N)\subset SL(N)$. The proof is done by considering the square of the antipode.

We refer the reader to Tanisaki [12] for the results on the representation theory of the quantized enveloping algebra, which we use below.

§2. Quantum groups.

Let G be a connected complex semisimple Lie group and let $\mathfrak{g}$ be its Lie algebra. Let $A=(a_{ij})_{1\leq i,j\leq l}$ be the Cartan matrix of $\mathfrak{g}$ and let $\boldsymbol{d}=(d_1,\cdots,d_l)$ be positive integers such that $d_ia_{ij}=d_ja_{ji}$. The quantized enveloping algebra $U_\hbar(\mathfrak{g})=U_{\hbar,\boldsymbol{d}}(\mathfrak{g})$ is the $\mathbf{C}[[\hbar]]$-algebra which is $\hbar$-adically generated by elements X_i, Y_i, H_i $(1\leq i\leq l)$ satisfying the following fundamental relations:

$$H_iH_j=H_jH_i\,,$$

$$H_iX_j-X_jH_i=a_{ij}X_j\,,\quad H_iY_j-Y_jH_i=-a_{ij}Y_j\,,$$

$$X_iY_j-Y_jX_i=\delta_{ij}\frac{K_i-K_i^{-1}}{q_i-q_i^{-1}}\,,$$

$$\sum_{0\leq n\leq 1-a_{ij}}(-1)^n\begin{bmatrix}1-a_{ij}\\ n\end{bmatrix}_{q_i}X_i^{1-a_{ij}-n}X_jX_i^n=0\qquad(i\neq j)\,,$$

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