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Minimal Hypersurfaces Foliated by Geodesics of 4-Dimensional Space Forms

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§0. Introduction.

Minimal surfaces of a 3-dimensional Euclidean space have been studied by many researchers. One of the most classic example of minimal surfaces is a helicoid. The helicoid is a ruled surface, i.e., a surface foliated by lines of $\boldsymbol{R}^3$. The following fact is well-known: minimal, ruled surface of $\boldsymbol{R}^3$ is either a part of a plane $\boldsymbol{R}^2$, or a part of the helicoid (cf. [1]). Barbosa-Dajczer-Jorge [2] generalize this theorem to the ruled minimal submanifolds of higher dimensional space forms.

In this paper, we determine minimal hypersurfaces M given by $M=\{\exp_p(t\xi)\,;\,p\in\Sigma,\ t\in\boldsymbol{R}\}$, where Σ is a minimal surface of constant curvature in a 4-dimensional space form $\tilde{M}$, and ξ is a (local) unit normal vector field on Σ. Such a minimal surface Σ is classified by Kenmotsu [5]. In §2, we find the equations for a surface Σ and a unit normal vector field ξ on Σ with respect to which $M=\{\exp_p(t\xi)\,;\,p\in\Sigma,\ t\in\boldsymbol{R}\}$ is minimal in $\tilde{M}$. In §3, §4, and §5, we solve the equations when Σ is totally geodesic in $\tilde{M}$, the minimal Clifford torus $S^1\times S^1\subset S^3\subset S^4$, and Σ is a Veronese surface of S^4, respectively. As a consequence, we find all minimal hypersurfaces M of S^4 satisfying the following conditions (theorem 5.1): (1) M contains a Veronese surface Σ of S^4, (2) M is foliated by great circles S^1 of S^4 which intersect Σ orthogonally, (3) the type number (i.e., the rank of the shape operator) of M is equal to 3 on some open set which intersects Σ. The proof is reduced to solving a differential equation of a holomorphic function.

Concerning this theorem, we note that minimal hypersurfaces with type number 2 of n-dimensional space forms ($n\geqq 4$) are investigated by Dajczer-Gromoll [4]. In fact, such a minimal hypersurface is obtained by the image of a minimal surface under the Gauss map. But it seems that little is known about minimal hypersurfaces of S^4 with type number 3, other than the generalized Clifford torus $S^2\times S^1$ (cf. [6], [7]).

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