

Initial Boundary Value Problem for the Wave Equation in a Domain with a Corner

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Introduction.

In this paper, we consider the mixed problem for the wave equation in the domains $\{(t, x, y, w) \mid t > 0, x > 0, y > 0, w \in \mathbb{R}^n\}$ and $\{(t, x, y, z, w) \mid t > 0, x > 0, y > 0, z > 0, w \in \mathbb{R}^n\}$. On the boundary $x=0$, an oblique boundary condition is given and on the other boundary ($y=0$ or $z=0$), the Dirichlet or Neumann boundary condition is given. Such a problem was considered in [1], [2], [5], [14], [16] and [17].

The aim of this paper is to give the energy inequality for the above problem with non-homogeneous boundary condition, with the aid of which we can prove an existence and uniqueness theorem. Our result is an extension of the result in [1], [16] and [17]. This energy estimate is the same as the one for the mixed problem in the domain with smooth boundary. The similar result is obtained only in [16] and [17].

Our method is to reduce the mixed problem for the wave equation in a domain with a corner to the one for symmetric hyperbolic systems of first order with positive (or non-negative) boundary condition on the boundaries. This method is discovered in [27], and is given in Appendix. Such a method is treated and developed in [18], [7], [19], [20], [22], [24] and [26], and used in [17], [23] and [25]. In [27], we improved the results in [19: §4] and [26: §3], and obtain our results by the use of its reformation in [27].

As for the other results on the mixed problem for hyperbolic equations in a domain with non-smooth boundary, we can refer to [6], [13], [15] and [21].

As for the other symmetrization on the Cauchy problem and the mixed problem for hyperbolic equation, we can refer to [3], [4], [8], [9], [10] and [11].

An outline of this paper is as follows. In §1, we give the notations. In §2, we state the problems and the results. In §3, we discuss the symmetrization of the mixed problems (P.1), (P.2) and (P.3). In §4, we treat the boundary estimate of the solution. In §5, we prove Main Theorem 1. In §6, we prove Main Theorem 2. In §7, we prove Main