TOKYO J. MATH.
VOL. 16, NO. 1, 1993

On Certain Affect-Free Equations

Kenzo KOMATSU

Keio University

§1. Affect-free equations.

What is the *simplest* affect-free equation (§5) of given degree n? Although many affect-free equations are known, simple examples are rare (cf. [5], [6]). Perhaps one of the simplest examples is the equation

$$x^n - x - 1 = 0\,, \tag{1.1}$$

which is affect-free for every $n > 1$ ([4], Theorem 4). Another possible answer to our question is the equation

$$x^n + 2x + 2 = 0\,, \tag{1.2}$$

which is also affect-free for every $n > 1$ (§4).

The equation (1.2) is much different from (1.1). For example, it is obvious that the left-hand side of (1.2) is irreducible; it is not at all obvious that the left-hand side of (1.1) is irreducible (Selmer [7]). Let α denote a root of (1.2), and let β denote a root of (1.1). Then the prime number 2 is completely ramified (§5) in $\boldsymbol{Q}(\alpha)$, whereas no prime numbers are completely ramified in $\boldsymbol{Q}(\beta)$ if $n > 2$. The discriminant of $\boldsymbol{Q}(\beta)$ is square-free ([4], Theorem 3), whereas the discriminant of $\boldsymbol{Q}(\alpha)$ is not square-free. Therefore, Theorem 1 of our previous paper [4] is not applicable to the equation (1.2).

The main purpose of the present paper is to prove the following theorem.

THEOREM 1. *Let* $a_0, a_1, \cdots, a_{n-1}$ $(n > 1)$ *be integers such that*

$$f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

is irreducible over $\boldsymbol{Q}$. *Let* α *be a root of* $f(x) = 0$, *and let* $K = \boldsymbol{Q}(\alpha)$, $\delta = f'(\alpha)$, $D = \operatorname{norm}\delta$ *(in* K*). Let* $x_0, x_1, \cdots, x_{n-1}$ *be integers such that*

$$D/\delta = x_0 + x_1\alpha + \cdots + x_{n-1}\alpha^{n-1}\,.$$

Suppose that the following three conditions are satisfied.

Received March 5, 1991