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Modified Jacobi-Perron Algorithm and Generating Markov Partitions for Special Hyperbolic Toral Automorphisms

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0. Introduction.

The fact that the boundaries of Markov partitions of hyperbolic toral automorphism on T^3 are not smooth was pointed out by Bowen [2]. Using the generating method of fractal curves by Dekking [3], T. Bedford gave Markov partitions with fractal boundaries on a suitable subclass of hyperbolic toral automorphism on T^3.

THEOREM (T. Bedford). *Let us assume that the* 3×3 *integral matrix* B *satisfies the following properties*:

(1) *B is non-negative and* $\det B=1$,

(2) *the maximum eigenvalue λ_0 of B is a Pisot number, that is, B has a single real expanding eigen value $\lambda_0>1$ and double contracting eigenvalues λ_1, λ_2 $(1>|\lambda_i|>0)$.*

Then there exists a Markov partition of toral automorphism T_B on T^3 with structure matrix ${}^tB$.

His main idea is to construct a bounded domain X and a partition $\{X_i: i=1, 2, 3\}$ on the expanding invariant plane $\mathbb{P}$ with respect to a linear map L_B^{-1} which induces the Markov endomorphism on X with structure matrix B by using the generating method of fractal curves. The purpose of this paper is to study more precisely the generating method of these domains. For this purpose, we introduce the concept of tilings of $\mathbb{P}$ by three kinds of parallelograms and the concept of the substitution Σ on the configurations of parallelograms. (See Figure 1.)

Using this idea, on the special class for B such that $B=A_1A_2\cdots A_k$, where

$$A_i\in\left\{\begin{pmatrix} a & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}, \begin{pmatrix} a & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix};\quad a\in N\right\},$$

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