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On the Convergence of the Spectrum of Perron-Frobenius Operators

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1. Introduction.

We will consider $\{F_t\}_{t=1,2,\cdots,\infty}$ a family of piecewise C^2 mappings from an interval I into itself. We denote by P_t the Perron-Frobenius operator corresponding to F_t:

$$\int_I P_t f(x)g(x)dx = \int_I f(x)g(F_t(x))dx \qquad \text{for } f \in L^1 \text{ and } g \in L^\infty ,$$

where we denote by L^1 (resp. L^∞) the set of integrable functions (resp. the set of bounded measurable functions). We denote by $Spec(F_t)$ the spectrum of P_t restricted to BV, the set of bounded functions. Here, as usual, we consider BV as a subset of L^1 by taking L^1-version and the norm

$$V(f) = \inf\{\text{the total variation of } \tilde{f} : \tilde{f} \text{ is a } L^1\text{-version of } f\} + \int_I |f(x)|dx .$$

We assume that F_t converges to F_∞ in piecewise C^1 (the definition will be stated in §2). In this situation, though P_t converges to P_∞ in L^1, P_t does not necessarily converge to P_∞ in BV. This means that general perturbation theories cannot be applied. Nevertheless, using Fredholm matrix which is defined in [10], our main theorem (Theorem A) states that $Spec(F_\infty)$ can be approximated by $Spec(F_t)$.

THEOREM A. *Assume that*

(1) *each* F_t *is a piecewise* C^2 *mapping with positive lower Lyapunov number* ξ_t $(t=1, 2, \cdots, \infty)$,

(2) F_t *converges to* F_∞ *in piecewise* C^1.

Then for z_∞ *which satisfies* $|z_\infty| < e^{\xi_\infty}$, $z_\infty^{-1} \in Spec(F_\infty)$ *if and only if there exists a sequence* $\{z_t\}$ *such that* z_t *converges to* z_∞ *and* $z_t^{-1} \in Spec(F_t)$.

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