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Equivariant Harmonic Maps Associated to Large Group Actions

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§1. Introduction.

For compact Riemannian manifolds (M, g) and (N, h), harmonic maps between them are critical points of the energy functional

$$E(\phi)=\frac{1}{2}\int_M |d\phi|^2 dv_g\,,$$

on the space of all smooth maps ϕ of M into N. Namely, for any smooth variation of ϕ, ϕ_t, $-\varepsilon<t<\varepsilon$, with $\phi_0=\phi$, it holds that

$$\frac{d}{dt}\bigg|_{t=0} E(\phi_t)=0\,.$$

A remarkable existence result, in compact case, is due to Eells-Sampson [7]. They showed that if the target manifold (N, h) has non-positive curvature, then there exists a minimizing harmonic map in its homotopy class for any smooth map of (M, g) into (N, h).

On the other hand, the notion of harmonic maps is well-defined for non-compact Riemannian manifolds, and existence problem is also interesting. Recently Li-Tam [10] showed the existence of a harmonic map from (M, g) into (N, h), provided that these manifolds are complete and have some curvature conditions. As an application, they investigated the boundary value problem of harmonic map between the hyperbolic spaces and showed the existence of one which is equal to a given C^3 boundary map with non zero energy density (see also Akutagawa [1]).

But so far results of explicit construction of harmonic maps between non-compact Riemannian manifolds are very few, except works by Baird [3] and Kasue and Washio [9]. Baird [3] reduced the harmonic map equation to an ordinary differential equation defined on $(0, \infty)$, and investigated the behavior of a solution at the origin. Kasue and

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