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A Study of Ordinary Differential Equations Arising from Equivariant Harmonic Maps

Keisuke UENO

Tôhoku University

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1. Introduction.

In the theory of harmonic maps between Riemannian manifolds, the problems of existence and construction are basic and important. One of the methods of constructing harmonic maps is the one by making use of ordinary differential equations. For example, Ding [2], Eells and Ratto [3], Ratto [5] and Smith [6] use the join of two maps and derive ordinary differential equations. They construct harmonic maps between spheres or between spheres and ellipsoids. Xin [10] studies equivariant harmonic maps with respect to Riemannian submersion. He applies this method to the existence of harmonic representatives of $\pi_{2m+1}(S^{2m+1})$([11]). Also Urakawa [8], Urakawa and author [9] investigate the theory of equivariant harmonic maps between Riemannian manifolds admitting large isometry group actions.

For constructing equivariant harmonic maps, it is important to study ordinary differential equations with singularities. In particular, in relation with the regularity of solutions and the problem asking how the image of equivariant harmonic maps expands, we want to know the asymptotic behavior of a solution of the ordinary differential equation nearby its singularities. On the other hand, from the viewpoint of ordinary differential equation theory it is interesting to investigate a solution on the blow-up phenomena or its regularity of a solution at singularities.

In this paper, we study the existence of a positive solution satisfying the following equations (1.1) and (1.2):

$$\begin{aligned}(1.1)\qquad &\ddot{r}(t)+\left\{p\frac{\dot{f}_1(t)}{f_1(t)}+q\frac{\dot{f}_2(t)}{f_2(t)}\right\}\dot{r}(t)\\ &-\left\{\mu^2\frac{h_1(r(t))h'_1(r(t))}{f_1(t)^2}+\nu^2\frac{h_2(r(t))h'_2(r(t))}{f_2(t)^2}\right\}=0 \qquad \text{on} \quad [0,\infty)\,;\end{aligned}$$

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