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On the Existence and Conformal Equivalence of Extremal Maps of Various Types

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1. Introduction.

Let (M, g) and (N, h) be compact smooth Riemannian manifolds of dimension m and n, respectively and $f: M \to N$ be a smooth map. As a natural generalization of a harmonic map, an exponentially harmonic map and a p-harmonic map $(1 < p < \infty)$ are defined by extremals of the functionals $\int_M e^{|df|^2/2} v_g$ and $\int_M |df|^p v_g$, respectively. And a $(1, p)$-harmonic map is defined by an extremal of the less degenerate functional $\int_M (1 + |df|^2)^{p/2} v_g$ as in [D-F]. We call them extremal maps of various types.

We will study the following problem; for a given smooth map $\varphi: M \to N$, does there exist an extremal map f such that f is homotopic to φ?

Eells and Ferreira [E-F] gave some positive answer to the above problem allowing conformal change of the Riemannian metric g in the case of harmonic maps. Hong [H] also solved in the case of exponentially harmonic, by the method of the conformal equivalence of harmonic maps and exponentially harmonic maps. On the other hand, there are some results due to H. Takeuchi [T] about the conformal equivalence of p-harmonic maps and p'-harmonic for some p and p'.

In this paper, we give a positive answer to the above problem allowing conformal change of g in the case of $(1, p)$-harmonic for any p $(1 < p < \infty)$, and obtain conformal equivalence among extremal maps of various types each others.

We also study their stability under the conformal change of g in the last section.

Now, we will illustrate our results in this paper as the following diagram.

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