TOKYO J. MATH.
VOL. 19, No. 1, 1996

On the A. Beurling Convolution Algebra

Kazuo ANZAI, Kenji HORIE and Sumiyuki KOIZUMI

Kagawa University, Takamatu National College of Technology and Keio University
(Communicated by M. Maejima)

1. A. Beurling [2] considered a class of functions of $\mathbf{R}^1$, each member of which is the Fourier transform of an integrable function. The purpose of this paper is to extend his results to the class of functions on $\mathbf{R}^n$. Let us start to set notations, definitions and theorems, which we shall ask for, according to A. Beurling [2]. We consider a normed family Ω of strictly positive functions $\omega(x)$ on $\mathbf{R}^n$ which are measurable with respect to the ordinary Lebesgue measure dx, and furthermore, together with the norm $N(\omega)$, satisfy the following conditions:

(I) For each $\omega\in\Omega$, $N(\omega)$ takes a finite value,

$$0<\int\omega dx\le N(\omega)\,. \tag{1.1}$$

(II) If λ is a positive number and $\omega\in\Omega$, then $\lambda\omega\in\Omega$ and

$$N(\lambda\omega)=\lambda N(\omega)\,. \tag{1.2}$$

(III) If $\omega_1,\omega_2\in\Omega$, the sum $\omega_1+\omega_2$ as well as the convolution $\omega_1*\omega_2$ are also in Ω and

$$N(\omega_1+\omega_2)\le N(\omega_1)+N(\omega_2)\,, \tag{1.3}$$

$$N(\omega_1*\omega_2)\le N(\omega_1)N(\omega_2)\,. \tag{1.4}$$

(IV) Ω is complete under the norm N in the sense that for any sequence $\{\omega_n\}_1^\infty\subset\Omega$ such that $\sum_1^\infty N(\omega_n)<\infty$, it is satisfied that $\omega=\sum_1^\infty\omega_n$ is in Ω and

$$N(\omega)\le\sum_1^\infty N(\omega_n)\,. \tag{1.5}$$

The set of measures $\{\omega dx;\omega\in\Omega\}$ constitutes our starting point for the following constructions of Banach algebra and shall be referred to as a normed semi-ring of

Received September 6, 1994
Revised October 13, 1995