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Cartan Embeddings of Compact Riemannian 3-Symmetric Spaces

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Dedicated to Professor Masaru Takeuchi on his sixtieth birthday

Introduction.

Let G be a compact connected Lie group and σ be an automorphism on G. We put $K=\{k\in G:\sigma(k)=k\}$. A mapping $g\mapsto g\sigma(g^{-1})$ of G into G naturally induces an embedding of G/K into G. We denote the embedding by Ψ_σ and call it the *Cartan embedding*.

If we assume that σ is an involutive automorphism, then Ψ_σ is a totally geodesic embedding. The author classified the compact irreducible symmetric pairs (G, K) such that the image of the corresponding Cartan embedding is a stable minimal submanifold of G ([4]).

In this paper, we study the similar problem for the case that G is a compact simple Lie group and σ is an automorphism of order 3. In this case, the image of the Cartan embedding is not necessarily a minimal submanifold. So we study

1. Is Cartan embedding a minimal embedding?
2. If it is a minimal embedding, then is the image a stable minimal submanifold?

1. Cartan embedding.

Let G be a compact connected simple Lie group and σ be an automorphism on G. We denote by $\mathfrak{g}$ and $\mathfrak{k}$ the Lie algebras of G and $K=\{k\in G:\sigma(k)=k\}$ respectively. Take an $Ad(G)$-invariant and $d\sigma$-invariant inner product $\langle\ ,\ \rangle$ on $\mathfrak{g}$. We extend $\langle\ ,\ \rangle$ to a biinvariant Riemannian metric on G and denote it also by $\langle\ ,\ \rangle$. Let $\mathfrak{m}$ be the orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$. We identify the subspace $\mathfrak{m}$ with the tangent space M_o of M at the origin $o=eK$ by the projection $G\to M=G/K$. A G-invariant Riemannian metric g on M is said to be a *normal homogeneous metric* if it is associated with the restriction

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