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An Ill-Posed Estimate of Positive Solutions of a Degenerate Nonlinear Parabolic Equation

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1. Introduction.

In this paper we shall consider the non-characteristic Cauchy problem for a degenerate nonlinear parabolic equation and derive an estimate of positive solutions in terms of the bounds on the Cauchy data and the solutions.

In [10] Pucci studied the non-characteristic Cauchy problem of the linear parabolic differential equation of second order, where he showed the estimate of continuous dependence for positive solutions. Cannon [2] removed the positivity of solutions and derived an estimate, which is not of continuous dependence. His result is as follows:

Suppose $u_t=u_{xx}$, $0<x<1$, $0<t<T$, $u(x,0)=0$, $0<x<1$, $|u(0,t)|$, $|u_x(0,t)|\le\varepsilon$, $0<t<T$, $|u(1,t)|\le M$, $0<t<T$. Then $|u(x,t)|\le M_1^{1-\beta(x)}\varepsilon^{\beta(x)}$, $0<x<1$, $0<t<T$, where $0<\beta(x)<1$ and M_1 depends on M.

The similar estimation was proved by Glagoleva [3] for more general linear parabolic equations. In this connection, there are referred to several contributions (see e.g., [13], [14]). From the estimate given there, we immediately see that the solution will vanish near the Cauchy surface, if the Cauchy data equals zero, particularly. Previously to these results Mizohata [8] established the uniqueness in non-characteristic Cauchy problem for linear parabolic equations.

In this way naturally arises the question: Does the above result hold too for nonlinear parabolic equations? Concerning this problem, Varin [15] has recently extended Glagoleva's work [3] to the case for a semilinear parabolic equation.

In this paper we shall consider the non-characteristic Cauchy problem for degenerate nonlinear parabolic equations and achieve an ill-posed estimate for positive solutions.

Our equation is of the form (2.1). But in the latter half we shall assume that the space dimension is one. Let $x=(x_1,\cdots,x_N)$ and $\Delta=\sum_{i=1}^N\partial_{x_i}^2$. There are many

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