TOKYO J. MATH.
VOL. 20, NO. 1, 1997

Realization of Spaces $E_6/(U(1)Spin(10))$, $E_7/(U(1)E_6)$, $E_8/(U(1)E_7)$ and Their Volumes

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(Communicated by T. Nagano)

Freudenthal [3], [4] introduced cross operations $\times$ on the spaces $\mathfrak{J}^C$, $\mathfrak{P}^C$ and $\mathfrak{e}_8{}^C$, respectively. Using these operations, we give definite expressions of the exceptional symmetric spaces *EIII*, *EVII* and the twister space $Z(EIX)$ of the exceptional symmetric space *EIX* as follows:

$$EIII = \{X \in \mathfrak{J}^C \mid X \times X = 0, X \neq 0\}/C^* \cong E_6/(U(1) \times Spin(10))/\mathbf{Z}_4 ,$$

$$EVII = \{P \in \mathfrak{P}^C \mid P \times P = 0, P \neq 0\}/C^* \cong E_7/(U(1) \times E_6)/\mathbf{Z}_3 ,$$

$$Z(EIX) = \{R \in \mathfrak{e}_8{}^C \mid R \times R = 0, R \neq 0\}/C^* \cong E_8/(U(1) \times E_7)/\mathbf{Z}_2 .$$

We then compute the first Chern classes by using the above expressions and calculate the volumes $\mu(M, g)$ of those spaces M with respect to the metric g indicated in 1.2, 2.2 and 3.2. This follows from the cohomology ring structures of the spaces together with the Borel-Hirzebruch formula. The results enable us calculating the volumes of the symmetric spaces of exceptional type [1]. Our results are as follows:

$$\mu(EIII, g) = \frac{78}{16!}\pi^{16} , \qquad \mu(EVII, g) = \frac{13110}{27!}\pi^{27} ,$$

$$\mu(Z(EIX), g) = \frac{2^{12}\ 3^2\ 5^2\ 7\ 31\ 37\ 41\ 43\ 47\ 53}{57!}\pi^{57} .$$

1. *EIII*.

1.1. The Lie group E_6 and its Lie algebra $\mathfrak{e}_6$. Let $\mathfrak{C}$ be the Cayley algebra and $\mathfrak{J} = \{X \in M(3, \mathfrak{C}) \mid X^* = X\}$ be the exceptional Jordan algebra with the Jordan multiplication $X \circ Y = \frac{1}{2}(XY + YX)$ and the Freudenthal multiplication

$$X \times Y = \tfrac{1}{2}(2X \circ Y - \mathrm{tr}(X)Y - \mathrm{tr}(Y)X + (\mathrm{tr}(X)\mathrm{tr}(Y) - (X, Y))E) .$$

Received June 23, 1995