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Extrinsic Hyperspheres of Naturally Reductive Homogeneous Spaces

Koji TOJO

Chiba University

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0. Introduction.

In [2], Chen investigated extrinsic spheres of (locally) symmetric spaces and obtained the following.

THEOREM. *Let N be an n-dimensional submanifold in a locally symmetric space $\tilde{M}$. Then N is an extrinsic sphere in $\tilde{M}$ if and only if N is an extrinsic hypersphere in a $(n+1)$-dimensional totally geodesic submanifold of constant sectional curvature.*

On the other hand the author has proved in [7] the following. Let G be a compact simple Lie group and K a closed subgroup of G. If the normal homogeneous space G/K contains a totally geodesic hypersurface N, then G/K is a space with constant sectional curvature. Then, in this paper, we treat a similar problem in case that G/K is a naturally reductive homogeneous space and N is an extrinsic hypersphere.

The paper is organized as follows. In Section 2 we write the Levi-Civita connections of homogeneous spaces in terms of the Lie algebra. In Section 3, using a result of Section 2, we shall describe circles of homogeneous spaces in terms of Lie algebras. Section 4 is devoted to prove the following theorem.

MAIN THEOREM. *If a naturally reductive homogeneous space G/K admits an extrinsic hypersphere, then G/K is a space with constant sectional curvature.*

1. Preliminaries.

In this section we recall some basic facts with respect to the Levi-Civita connection on Riemannian manifolds.

Let (M, g) be an n-dimensional Riemannian manifold and ∇ the Levi-Civita connection of (M, g). Let $\{e_1, \cdots, e_n\}$ be a local orthonormal frame field and $\{\omega^1, \cdots, \omega^n\}$

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