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The Sym-Bobenko Formula and Constant Mean Curvature Surfaces in Minkowski 3-Space

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1. Introduction.

Recently, Dorfmeister, Pedit and Wu discovered a Weierstrass-type representation for harmonic maps from a Riemann surface into symmetric spaces [DPW]. In their formula, the Weierstrass data are defined as meromorphic potentials, i.e. meromorphic 1-forms on a Riemann surface with values in an infinite-dimensional loop algebra. They regarded a harmonic map as a map taking values in a twisted loop group and showed that every harmonic map from a Riemann surface into a symmetric space is obtained by integrating the potential. In a related paper, Dorfmeister and Haak have constructed constant mean curvature surfaces by applying the Sym-Bobenko formula [DH] to the loop-group-valued maps given by integrating the potentials.

On the other hand, Kenmotsu discovered a representation formula for immersions with prescribed mean curvature from a simply connected Riemann surface into Euclidean 3-space. In particular, he obtained a formula for an immersion with constant mean curvature whose Gauss map is a given harmonic map [K]. And Akutagawa and Nishikawa constructed the Minkowski 3-space version of the above formula [AN].

Motivated by these results, the present paper has two aims. The first is to establish a natural correspondence between the following two spaces: the space of conformal spacelike immersions with constant mean curvature from a simply connected Riemann surface Σ into Minkowski 3-space, and that of nowhere anti-holomorphic harmonic maps from Σ into the Poincaré half plane, regarded as the riemannian symmetric space $SL(2, \mathbf{R})/SO(2)$. The second is to prove the Lorentzian version of the Sym-Bobenko formula and apply it to construct spacelike immersions with constant mean curvature.

In section 2 we shall first prepare notations used in the later sections and recall the identification of the riemannian symmetric space $SL(2, \mathbf{R})/SO(2)$ with the unit disk equipped with the Poincaré metric and also with the Poincaré half plane. In section 3 we shall define a $sl(2, \mathbf{R})$-valued 1-form A^f on a Riemann surface Σ associated to a

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