TOKYO J. MATH.
VOL. 20, NO. 2, 1997

On the Cyclotomic Unit Group and the p-Ideal Class Group of a Real Abelian Number Field

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(Communicated by T. Suzuki)

1. Introduction.

Let p be an odd prime number, which will be fixed throughout the present paper. For any real abelian number field K, let K_∞ denote the cyclotomic $\mathbf{Z}_p$-extension of K and K_n its n-th layer over K. Let A_n and $A'_n = A_n/(\langle$ideal classes of K_n which contain a prime ideal above $p\rangle \cap A_n)$ be the p-Sylow subgroups of the ideal class group and of the p-ideal class group, respectively, of K_n. Let E_n and C_n be the groups of units and of cyclotomic units in the sense of Sinnott, respectively, of K_n (cf. [7]). Denote by B_n the p-Sylow subgroup of the quotient group E_n/C_n. We write $\lambda_p(K)$ and $\mu_p(K)$ for the Iwasawa λ and μ invariants, respectively, of K_∞/K.

It is well known that the order of A_n and B_n are "almost" equal. For example, if $p \nmid [K : \mathbf{Q}]$ then $\sharp(A_n) = \sharp(B_n)$ (cf. [7]). Furthermore, the Iwasawa main conjecture proved by B. Mazur and A. Wiles implies that the characteristic ideals of $\mathbf{Z}_p[[\mathrm{Gal}(K_\infty/K)]]$-modules $\varprojlim A_n$ and $\varprojlim B_n$ coincide, where the projective limits are taken with respect to the norm maps (cf. [6], [3]). So it arises a natural question: Is there any deeper relation between the Galois module structures of A_n and B_n?

In the present paper, we shall give an answer to the above question under the assumption that Greenberg's conjecture (cf. [2]) is valid. Specifically, we shall prove the following:

THEOREM 1. *Let K be a real abelian number field with $p \nmid [K : \mathbf{Q}]$. If we assume that Greenberg's conjecture is valid for K and p, namely, that the Iwasawa invariants $\lambda_p(K)$ and $\mu_p(K)$ vanish, then A'_n is embedded in B_n as a Galois module (namely, $\mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q})$-module) for all sufficiently large n.*

We remark that $\mu_p(K)$ always vanishes in the above theorem by the Ferrero-Washington theorem (cf. [1]).

We shall prepare some results about the Galois cohomology groups of cyclotomic unit groups in section 2, and give the proof of Theorem 1 in section 3.

Received February 28, 1996
Revised March 28, 1996