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Dynamical System on Cantor Set

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1. Introduction.

We will consider Cantor sets generated by piecewise $C^{1+\gamma}$ transformations ($\gamma>0$). In this article, we only consider Markov cases. Non Markov (but piecewise linear) cases will be studied in [6]. A heuristic argument will also appear in that paper.

Let us denote $I=[0,1]$. We assume that there exists a finite set $\mathscr{A}$ of symbols, and a subinterval $\langle a\rangle\subset I$ corresponds to a symbol $a\in\mathscr{A}$, and

1. $\bigcup_{a\in\mathscr{A}}\langle a\rangle=I$,
2. $\langle a\rangle\cap\langle b\rangle=\varnothing$ if $a\neq b$.

Take a subset $\mathscr{A}_1\subset\mathscr{A}$, and we consider a mapping F from $\bigcup_{a\in\mathscr{A}_1}\langle a\rangle$ to I such that

1. F is monotone on each $\langle a\rangle$ and it can extend to $\overline{\langle a\rangle}$ in $C^{1+\gamma}$ ($\gamma>0$) (piecewise $C^{1+\gamma}$),
2. if $F(\langle a\rangle)\cap\langle b\rangle\neq\varnothing$ for a, $b\in\mathscr{A}_1$, then $\overline{F(\langle a\rangle)}\supset\langle b\rangle$ (Markov),
3. $\xi=\liminf_{n\to\infty}\frac{1}{n}\operatorname{ess\,inf}_{x\in I}\log|F^{n\prime}(x)|>0$ (expanding),
4. for each a, $b\in\mathscr{A}_1$, there exists n such that $\overline{F^n(\langle a\rangle)}\supset\langle b\rangle$ (irreducible),

where we denote the closure of a set J by $\bar{J}$. Note that from the above assumption, we get

$$\log\operatorname*{ess\,inf}_{x\in I}|F^{n\prime}(x)|>0$$

for some $n>0$. Here we denote by F^n the n-th iteration of F:

$$F^n(x)=\begin{cases}x & \text{if}\quad n=0\,,\\ F^{n-1}(F(x)) & \text{if}\quad n\geq 1\,.\end{cases}$$

Thus, hereafter we assume without loss of generality that

$$\xi_0=\log\operatorname*{ess\,inf}_{x\in I}|F'(x)|>0\,.$$

We will consider a set

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