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On Pseudoconvex Domains in $\mathbf{P}^n$

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To the memory of Minoru Tada and Nobuo Sasakura

1. Introduction.

Let $\Omega \subsetneqq \mathbf{P}^n$ be a locally pseudoconvex domain and denote, for $z \in \Omega$, by $\delta_\Omega(z)$ the distance between z and $\partial\Omega$ measured with respect to the Fubini-Study metric. It is known from the work of A. Takeuchi [7], that $-\log\delta_\Omega$ is strictly plurisubharmonic on all of Ω and, hence, Ω is Stein. Therefore, it is reasonable to try to generalize function theory to locally pseudoconvex domains in $\mathbf{P}^n$.

In this article we will consider two questions in this direction, namely:

1) Are there localization principles for the Bergman kernel function and the Bergman metric (with respect to the measure coming from the Fubini-Study metric) on a suitable class of such domains?

2) Does local hyperconvexity of pseudoconvex domains $\Omega \subset \mathbf{P}^n$ imply also their global hyper-convexity?

In order to formulate our results with respect to 1) we denote by $d\lambda_{FS}$ the Fubini-Study volume element on $\mathbf{P}^n$ and put

$$A^2(\Omega) := \left\{ f \in \mathcal{O}(\Omega) : \int_\Omega |f(z)|^2 d\lambda_{FS} < \infty \right\}.$$

This is a Hilbert space with respect to the inner product

$$(f, g) := \int_\Omega f(z)\overline{g(z)} d\lambda_{FS} .$$

Notice, that always $\mathbf{C} \subset A^2(\Omega)$, since the volume of Ω is finite. The space $A^2(\Omega)$ has a (possibly constant) reproducing kernel

$$K_\Omega(\cdot, \cdot) : \Omega \times \Omega \to \mathbf{C}$$

which, in this article, will be called the *Bergman kernel* of Ω. We denote by $K_\Omega(z) := K_\Omega(z, z)$

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