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On a Generalization of the Conjecture of Jeśmanowicz

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1. Introduction.

In 1956, Sierpiński [S1] showed that the equation $3^x+4^y=5^z$ has only the positive integral solution $(x, y, z)=(2, 2, 2)$. Jeśmanowicz [J] conjectured that if a, b, c are Pythagorean numbers, i.e. positive integers satisfying $a^2+b^2=c^2$, then the equation $a^x+b^y=c^z$ has only the positive integral solution $(x, y, z)=(2, 2, 2)$ (cf. [S2]). It has been verified that this conjecture holds for many other Pythagorean numbers (cf. Lu [Lu], Takakuwa and Asaeda [Ta1], [Ta2], Takakuwa [Ta3], [Ta4], Adachi [A], Le [Le]).

As an analogy to this conjecture, in Terai [Te1], [Te2], [Te3] and [Te4], we proposed the following conjecture and proved it under some conditions when $p=2$, $q=2$ and r is an odd prime.

CONJECTURE. *If a, b, c, p, q, r are fixed positive integers satisfying $a^p+b^q=c^r$ with $p, q, r\geq 2$ and $(a, b)=1$, then the Diophantine equation*

$$a^x+b^y=c^z$$

has only the positive integral solution $(x, y, z)=(p, q, r)$.

In Terai [Te5], using a lower bound for linear forms in two logarithms, due to Laurent, Mignotte and Nesterenko [LMN], we showed the following (cf. Terai [Te4]):

THEOREM A. *Let l be a prime $\equiv 3 \pmod 8 < 11908$. Let a, b, c be fixed positive integers satisfying $a^2+lb^2=c^r$ with $(a, b)=1$ and r odd ≥ 3. Suppose that*

$$a\equiv 3 \pmod 8\,, \quad 2\| b\,, \quad \left(\frac{b}{a}\right)=-1\,, \quad \left(\frac{l}{a}\right)=1 \quad \text{and} \quad a\geq \lambda b\,,$$

where $\left(\frac{*}{*}\right)$ denotes the Jacobi symbol and

$$\lambda=\sqrt{l}\left\{\exp\left(\frac{2}{\log l+3235}\right)-1\right\}^{-1/2}.$$

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