TOKYO J. MATH.
VOL. 28, NO. 2, 2005

On Some Tubes over J-holomorphic Curves in S^6

Dedicated to Professor Koichi Ogiue on his 60th birthday

Hideya HASHIMOTO and Katsuya MASHIMO

Meijo University and Tokyo University of Agriculture and Technology

(Communicated by Y. Ohnita)

Introduction

Let (M, g) be a Riemannian manifold. We denote by $G^p(T_m M)$ the Grassmann manifold of all oriented p-dimensional linear subspaces of the tangent space $T_m M$ of M at $m \in M$ and by $G^p(TM)$ the Grassmann bundle $\bigcup_{m \in M} G^p(T_m M)$. Let V be a subbundle of $G^p(TM)$. A p-dimensional submanifold N of M is called a V*-submanifold* if $T_m N \in V$ holds for any $m \in N$. If a Lie group G acts on M, the action is naturally extended to the action of G on $G^p(TM)$. It seems to be an interesting problem to study V-submanifold for an orbit V of an action of G on $G^p(TM)$.

Let J be the standard almost complex structure of the 6-dimensional sphere S^6 and $\langle , \rangle$ the standard Riemannian metric. It is well-known that the group of automorphisms of $(S^6, J, \langle , \rangle)$ is isomorphic to the compact exceptional simple Lie group G_2. The complex volume form ω of the tangent space $T_m S^6$ at $m \in S^6$ is extended to a G_2-invariant (complex) 3-form on S^6. For a complex number κ ($|\kappa| \leq 1$), we put

$$V_\kappa = \{\xi \in G^3(TS^6) : \omega(\xi) = \kappa\} .$$

A 2-dimensional submanifold $\varphi : M^2 \to S^6$ is said to be a J-holomorphic curve if $J(d\varphi(T_m M)) = d\varphi(T_m M)$ holds for all $m \in M$. Bryant [1] showed that for any Riemann surface M there exists a superminimal J-holomorphic curve $\varphi : M \to S^6$ which has no geodesic point. In this note we study whether a tube over a J-holomorphic curve (in the direction of first or second) normal space is a V_κ-submanifold or not. In the case of tubes in the direction of second normal space, we shall prove the following

THEOREM 1. *Let $\varphi : M^2 \to S^6$ be a J-holomorphic curve without geodesic point. If a tube $\tilde{\varphi}_{2,\gamma}$ over φ of radius γ is a V_κ-submanifold, then one of the following holds*

(i) $\gamma = \pi/2$ *and* $\kappa = 1$,

Received June 22, 2004