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Spectral Geometry of Kähler Hypersurfaces in a Complex Grassmann Manifold

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1. Introduction

Let M be a compact C^∞-Riemannian manifold, $C^\infty(M)$ the space of all smooth functions on M, and Δ the Laplacian on M. Then Δ is a self-adjoint elliptic differential operator acting on $C^\infty(M)$, which has an infinite discrete sequence of eigenvalues:

$$Spec(M) = \{0 = \lambda_0 < \lambda_1 < \lambda_2 < \cdots < \lambda_k < \cdots \uparrow \infty\}.$$

Let $V_k = V_k(M)$ be the eigenspace of Δ corresponding to the k-th eigenvalue λ_k. Then V_k is finite-dimensional. We define an inner product $(\ ,\ )_{L^2}$ on $C^\infty(M)$ by

$$(f,\ g)_{L^2} = \int_M f g\, dv_M\,,$$

where dv_M denotes the volume element on M. Then $\sum_{t=0}^{\infty} V_t$ is dense in $C^\infty(M)$ and the decomposition is orthogonal with respect to the inner product $(\ ,\ )_{L^2}$. Thus we have

$$C^\infty(M) = \sum_{t=0}^{\infty} V_t(M) \quad (\text{in } L^2\text{-sense}).$$

Since M is compact, V_0 is the space of all constant functions which is 1-dimensional.

In this point of view, it is one of the simplest and the most interesting problems to estimate the first eigenvalue. In [13], A. Ros gave the following sharp upper bound for the first eigenvalue of Kähler submanifold of a complex projective space.

THEOREM 1.1. *Suppose that M is a complex m-dimensional compact Kähler submanifold of the complex projective space $\mathbf{C}P^n$ of constant holomorphic sectional curvature c. Then the first eigenvalue λ_1 satisfies*

$$\lambda_1 \leqq c(m+1).$$

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