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Some Results on Additive Number Theory IV

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§1. The main theorem.

Let $\omega(n)$ denote the number of distinct prime factors of a positive integer n.

THEOREM. *Let $\alpha<\beta$. Let $A(N; \alpha, \beta)$ denote, for sufficiently large positive integer N, the number of representations of N as the sum of the form $N=p+n$, where p is prime, and n is a positive integer such that*

$$\log\log N+\alpha\sqrt{\log\log N}<\omega(n)<\log\log N+\beta\sqrt{\log\log N}\ ,$$

then, as $N\to\infty$, we have

$$A(N; \alpha, \beta)\sim\frac{N}{\log N}\cdot\frac{1}{\sqrt{2\pi}}\int_\alpha^\beta e^{-x^2/2}dx\ .$$

We shall give a proof of this theorem in section 2. Our proof runs in the same lines as in my paper [6], but it uses also Bombieri's mean value theorem and Brun-Titchmarsh's inequality. It is to be noticed that somewhat analogous theorem was proved in Halberstam [3] using Siegel-Walfisz's theorem. It might perhaps be possible to prove our theorem in a similar style as in [3], but I hope that it would be of interest to prove the theorem in our way.

As was shown in Gallagher [2], Bombieri's theorem can be deduced rather simply from Siegel-Walfisz's theorem, and is far more conveniently applicable in our situation. For Bombieri's theorem cf. Bombieri [1], Gallagher [2], Halberstam-Richert [4], p. 111, Mitsui [5], Chap. 8.

We shall shorten the paper by omitting the similar parts of the proof as in Tanaka [6].

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