TOKYO J. MATH.
VOL. 2, No. 1, 1979

The Class Group of the Rees Algebras over Polynomial Rings

Yasuhiro SHIMODA

Tokyo Metropolitan University

Introduction

Let A be a commutative ring with unit element and let $A[X]$ denote a polynomial ring over A with an indeterminate X. For an ideal $\mathfrak{a}$ of A we put $\mathscr{R}(A, \mathfrak{a})=A[\{aX; a\in\mathfrak{a}\}, X^{-1}]$, the A-subalgebra of $A[X, X^{-1}]$ generated by $\{aX; a\in\mathfrak{a}\}$ and X^{-1}, and we call it the Rees algebra of $\mathfrak{a}$ over A.

$\mathscr{R}(A, \mathfrak{a})$ is a graded subring of $A[X, X^{-1}]$, whose graduation is given by $\mathscr{R}_n(A, \mathfrak{a})=\mathfrak{a}^n X^n$ for $n\geqq 0$ and $\mathscr{R}_n(A, \mathfrak{a})=A$ for $n<0$. Note that $\mathscr{R}(A, \mathfrak{a})$ is canonically identified with the ring $\bigoplus_{n\in Z}\mathfrak{a}^n$ where $\mathfrak{a}^n=A$ for $n<0$.

The aim of this paper is to prove the following theorem.

THEOREM. *Let k be a Krull domain and let $W_1, \cdots, W_s$ be indeterminates over k. Then, for every positive integer n, $\mathscr{R}(k[W_1, \cdots, W_s], (W_1, \cdots, W_s)^n)$ is a Krull domain and $C(\mathscr{R})=C(k)\oplus Z/nZ$. (Here $C(\cdot)$ denotes the divisor class group.)*

By the theorem we have the following result immediately.

COROLLARY. *If k is a field, then $\mathscr{R}(k[W_1, \cdots, W_s], (W_1, \cdots, W_s)^n)$ is a Macaulay normal domain and $C(\mathscr{R})=Z/nZ$.*

§1. Proof of Theorem.

Let k, $W_1, \cdots, W_s$, n be as in the introduction and let $X^{-1}=U$. We denote $\mathscr{R}(k[W_1, \cdots, W_s], (W_1, \cdots, W_s)^n)$ by T. Let Λ_n be the set of the indexes $(\alpha)=(\alpha_1, \cdots, \alpha_s)$ where α_i's are nonnegative integers with $\sum_{j=1}^{s}\alpha_j=n$ and let $R=k[W_1, \cdots, W_s, U]$. Then $T=k[W_1, \cdots, W_s, U, \{W^{(\alpha)}/U\}_{\alpha\in\Lambda_n}]$ and T is a k-subalgebra of $R[X]$, where $W^{(\alpha)}$ denotes $W_1^{\alpha_1}\cdots W_s^{\alpha_s}$.

Received June 22, 1978