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Analytic Continuation of Arithmetic Holomorphic Functions on a Half Plane

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Introduction

The entire arithmetic functions of one variable have been studied by many mathematicians. For example, see R. Boas [3] and R. Buck [4]. Recently V. Avanissian and R. Gay [1] studied entire arithmetic functions of exponential type of n variables using the theory of analytic functionals. In this paper we consider the arithmetic holomorphic functions on a half plane using the theory of analytic functional with non-compact carrier. We will obtain a sufficient condition for an arithmetic holomorphic function to be entire.

§1. Analytic functionals with non-compact carrier.

In this section we recall the definition of analytic functional with non-compact carrier. Let L be the closed half strip in the complex plane:

$$L=\{z=x+iy;\ x\geqq a,\ |y|\leqq k\}\ ,\quad i=\sqrt{-1}\ .$$

By L_ε we denote the ε-neighborhood of L:

$$L_\varepsilon=L+[-\varepsilon,\ \varepsilon]+i[-\varepsilon,\ \varepsilon]\ .$$

For $\varepsilon>0$, $\varepsilon'>0$ and $0\leqq k'<1$, we define the function space $Q_b(L_\varepsilon;\ k'+\varepsilon')$ as follows:

$$Q_b(L_\varepsilon;\ k'+\varepsilon')=\left\{f\in\mathcal{O}(\operatorname{int}L_\varepsilon)\cap C(L_\varepsilon);\ \sup_{z\in L_\varepsilon}|f(z)\exp((k'+\varepsilon')z)|<+\infty\right\}$$

where $\mathcal{O}(\operatorname{int}L_\varepsilon)$ denotes the space of holomorphic functions on the interior

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