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A Necessary Condition for Hypoellipticity of Degenerate Elliptic-Parabolic Operators

Kazuo AMANO

Tokyo Metropolitan University
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Introduction

The aim of this paper is to study hypoellipticity of degenerate elliptic-parabolic operators from the view point of the control theory. Hörmander and Oleĭnik-Radkevič proved (see [4]) that the degenerate elliptic-parabolic operator

$$L=\sum_{i,j=1}^{d}a_{ij}(x)\frac{\partial^2}{\partial x_i\partial x_j}+\sum_{i=1}^{d}b_i(x)\frac{\partial}{\partial x_i}+c(x) \tag{1}$$

in an open set M in $\boldsymbol{R}^d$ with real C^∞-smooth coefficients is hypoelliptic if $\dim\mathscr{L}(X_0, X_1, \cdots, X_d)\equiv d$ (for the notation, see §1), where

$$\begin{aligned} X_0&=\sum_{i=1}^{d}\Big(b_i-\sum_{j=1}^{d}\frac{\partial a_{ij}}{\partial x_j}\Big)\frac{\partial}{\partial x_i}\ , \\ X_i&=\sum_{j=1}^{d}a_{ij}\frac{\partial}{\partial x_j}\ , \qquad 1\leqq i\leqq d\ , \end{aligned} \tag{2}$$

and conversely, when the coefficients are real analytic, $\dim\mathscr{L}(X_0, X_1, \cdots, X_d)\equiv d$ if the operator L is hypoelliptic. Chow and Nagano proved (see [7]) that for a set of C^∞-smooth vector fields $\{X_0, X_1, \cdots, X_d\}$ the system

$$\dot{x}=\sum_{i=0}^{d}\xi_i X_i(x)\ , \qquad \xi_i\in\boldsymbol{R}^1 \tag{3}$$

is controllable in every subdomain in M if $\dim\mathscr{L}(X_0, X_1, \cdots, X_d)\equiv d$, and proved that the converse proposition holds when the vector fields are real analytic. Thus we are led naturally to the following problems:

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