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On the Duality Mapping of l^∞

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This paper is concerned with a measure theoretic characterization of the duality mapping of the space l^∞ of bounded sequences of real numbers. The duality mapping of a Banach space X is a possibly multi-valued mapping F from X into its dual space X^* which assigns to each $u \in X$ a subset of X^* defined by

$$F(u)=\{f \in X^*: \langle u, f\rangle=\|u\|^2=\|f\|^2\} ,$$

where $\langle u, f\rangle$ stands for the value of $f \in X^*$ at the point $u \in X$. The mapping F is well-defined on all of X by means of the Hahn-Banach theorem, and it is well-known ([1], [4], [9]) that $F(u)$ is weakly-star compact and convex for each $u \in X$; and F is weakly-star demi-closed in the sense that if u_n converges strongly to u in X, $f_n \in F(u_n)$, and f is a weak-star cluster point of the sequence $\{f_n: n\uparrow\infty\}$, then $f \in F(u)$. The space l^∞ is one of the typical non-reflexive classical Banach spaces in the sense that it is a Banach lattice with respect to the usual ordering and every separable Banach space can be embedded isometrically and isomorphically in l^∞. Accordingly, the duality mapping of l^∞ is a prototype of the duality mappings of general non-reflexive Banach spaces.

Here we investigate the structure and topological properties of the duality mapping F of l^∞. This problem was arised both in the study of generalized derivatives of strongly absolutely continuous functions which take values in non-reflexive Banach spaces and in the investigation of nonlinear dissipative operators. The results obtained in this paper will suggest not only typical properties possessed by the duality mapping of a general nonreflexive Banach space but also counterexamples concerning generalized derivatives and nonlinear dissipative operators.

Our work is mainly devoted to two problems: The first aim is to investigate the structure of the values $F(u)$, $u \in l^\infty$; and the second

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