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Minimal Immersions of Riemannian Products into Real Space Forms

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Introduction

Let M_1(resp. M_2) be an m(resp. $n-m$)-dimensional Riemannian manifold. An isometric immersion f of a Riemannian product $M_1\times M_2$ into an $(n+p)$-dimensional Euclidean space R^{n+p} is called a product immersion if there is an orthogonal product decomposition $R^{n+p}=R^{n_1}\times R^{n_2}$ together with isometric immersions f_1: $M_1\to R^{n_1}$ and f_2: $M_2\to R^{n_2}$ such that $f=f_1\times f_2$. Furthermore an isometric immersion g of a Riemannian product $M_1\times M_2$ into an $(n+p)$-dimensional sphere $S^{n+p}(r)$ with radius r in R^{n+p+1} is called a product immersion if g is a product immersion of $M_1\times M_2$ into R^{n+p+1}. S. B. Alexander [1] and J. Moore [4] obtained some conditions for an immersion of a Riemannian product into Euclidean space to be a product immersion. On the other hand, K. Yano and S. Ishihara [7] determined compact orientable submanifolds with nonnegative sectional curvature immersed into a unit sphere whose mean curvature vectors are parallel and normal connections are trivial. These are products of spheres and these immersions are product immersions into the unit sphere. In this note, we shall investigate Riemannian products minimally immersed into a real space form and prove some theorems.

THEOREM. *A minimal submanifold of a hyperbolic space is irreducible. A minimal immersion of a Riemannian product into Euclidean space is a product of minimal immersions.*

THEOREM. *Let M_1(resp. M_2) be an m(resp. $n-m$)-dimensional compact orientable Riemannian manifold and M the Riemannian product of M_1 and M_2 minimally immersed into $(n+p)$-dimensional unit sphere. Then we have an integral inequality*

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