

## Some Remarks on Subvarieties of Hopf Manifolds

Masahide KATO

*Sophia University*

### Introduction

A holomorphic automorphism  $g$  of a complex space  $\mathfrak{X}$  is called a *contraction* to a point  $O \in \mathfrak{X}$  if  $g$  satisfies the following three conditions:

- (i)  $g(O) = O$ ,
- (ii)  $\lim_{\nu \rightarrow +\infty} g^\nu(x) = O$  for any point  $x \in \mathfrak{X}$ ,
- (iii) for any small neighborhood  $U$  of  $O$  in  $\mathfrak{X}$ , there exists an integer  $\nu_0$  such that  $g^\nu(U) \subset U$  for all  $\nu \geq \nu_0$ ,

where  $g^\nu$  is the  $\nu$ -times composite of  $g$ . By [2]\*, the complex space  $\mathfrak{X}$  which admits a contracting automorphism is holomorphically isomorphic to an algebraic subset of  $C^N$  for some  $N$ . We identify  $\mathfrak{X}$  to the algebraic subset of  $C^N$ . Then there exists a contracting automorphism  $\tilde{g}$  of  $C^N$  to the origin  $O$  such that  $\tilde{g}|_{\mathfrak{X}} = g$  ([2], [3]). Obviously the action of  $\tilde{g}$  on  $C^N - \{O\}$  is free and properly discontinuous. Hence the quotient space  $H = C^N - \{O\} / \langle \tilde{g} \rangle$  is a compact complex manifold which is called a *primary Hopf manifold*. Sometimes we indicate by  $H^N$  an  $N$ -dimensional primary Hopf manifold. The compact complex space  $\mathfrak{X} - \{O\} / \langle g \rangle$  is clearly an analytic subset of a primary Hopf manifold. A compact complex manifold  $X$  of dimension  $n$  ( $n \geq 2$ ) is called a *Hopf manifold* if its universal covering is holomorphically isomorphic to  $C^n - \{O\}$  (Kodaira [4]).

The purpose of this paper is to show several properties of subvarieties of Hopf manifolds.

### §1. Hopf manifolds.

The following proposition shows that it is sufficient to consider only subvarieties of primary Hopf manifolds.

**PROPOSITION 1.** *Any Hopf manifold is a submanifold of a (higher dimensional) primary Hopf manifold.*

Received June 30, 1978

\* In [2], the condition (iii) is forgotten.