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Minimal Models in Proper Birational Geometry

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Introduction

In classical algebraic geometry, the following theorem due to Castelnuovo-Enriques-Zariski is fundamental, [9].

THEOREM A. *A non-singular projective surface S is minimal if S is relatively minimal and if S is not a ruled surface.*

In view of Enriques' criterion on ruled surfaces, the condition that S is not ruled may be replaced by $\kappa(S)\geqq 0$. Here, $\kappa(S)$ denotes the Kodaira dimension of S. Thus, we obtain

THEOREM B. *A non-singular projective surface S is minimal if S is relatively minimal surface with $\kappa(S)\geqq 0$.*

In this paper we shall consider analogues of the above facts in *proper birational geometry*. The category in which we shall work is that of schemes over the field of complex numbers $\boldsymbol{C}$.

In place of birational morphism and birational map in the classical theory, we shall use *proper birational morphism* and *strictly birational map* or *proper birational map*, respectively (see [2]). Thus for open surfaces, we shall define the concepts of *relatively minimal surface* and *minimal surface*. Using the notion of logarithmic Kodaira dimension we shall establish a theorem analogous to Theorem B (Theorem 1). Moreover, the notion of *∂-manifold* $(\bar{V}, D)$ will be introduced which consists of a non-singular complete algebraic variety $\bar{V}$ and a divisor with normal crossings D on $\bar{V}$. We shall study algebraic geometry for ∂-manifolds. The notions of relatively ∂-minimal model and properly ∂-minimal or ∂-minimal model will be introduced. For a ∂-surface $(\bar{S}, D)$ with $\bar{\kappa}(\bar{S}-D)=2$, an analogue of Theorem B will be established (Theorem 2).

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