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The Riemann-Hilbert Problem and its Application to Analytic Functions of Several Complex Variables

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Introduction

In this paper we shall prove the local existence of holomorphic functions in an analytic cover (a ramified Riemann domain) $\pi: Y \to X$ by using a solution of the Riemann-Hilbert problem (see §6). The existence of such functions was earlier proved in 1958 by H. Grauert and R. Remmert [10] and in 1960 by R. Kawai [11] by different methods. We can consider the functions on Y as many-valued functions on X which may have the branch points along the critical locus D of the analytic cover $\pi: Y \to X$. We shall construct such many-valued functions on X from the solutions of the total differential equation (1.1) whose monodromy representation is the one associated with the analytic cover $\pi: Y \to X$ (see §5). For this purpose, in §3, using the results of P. Deligne [6], we solve the Riemann-Hilbert problem in the following situation; let X be a connected Stein manifold and let D be a divisor of X (*not necessarily normal crossing*). Suppose that a representation ρ of $\pi_1(X-D, x_0)$ in $\mathrm{GL}_q(C)$ is given. We shall construct a total differential equation (1.1) whose monodromy is the given ρ. We can study in detail the case of $\dim X=2$ than that of $\dim X \geqq 3$, more precisely, when $\dim X=2$, if $H^2(X, Z)=0$, we can solve the Riemann-Hilbert problem *without apparent singularities* (Theorem 3). As an application of Proposition 2 of §3, we shall give a remark to the Riemann-Hilbert problem *in the restricted sense*, when X is a two-dimensional connected complex manifold. This problem was treated by K. Aomoto [1] by different method when X is an n-dimensional complex projective space (see §4). In solving the Riemann-Hilbert problem, we do not use the existence of resolution of X satisfying the condition that the inverse image of D is normal crossing, but we use essentially the extension theorems of coherent analytic sheaves of J.-P. Serre [15] and Y.-T. Siu

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