TOKYO J. MATH.
VOL. 3, NO. 2, 1980

Some Remarks on Quasi-Linear Evolution Equations in Banach Spaces

Nobuhiro SANEKATA

Okayama University
(Communicated by H. Sunouchi)

Introduction

This paper is concerned with the abstract Cauchy problem for a nonlinear equation of evolution of the form

$$(E) \qquad (d/dt)u(t)+A(t, u(t))u(t)=f(t, u(t)) , \quad \text{a.e.} \quad t\in[0, T] ,$$

with initial condition $u(0) = a$. Here u represents an unknown function taking its values $u(t)$ in a Banach space X; $A(t, y)\cdot$ is a linear operator in X depending on t in $[0, T]$ and y in a certain subset of X and f is a nonliear operator from a certain domain of $[0, T]\times X$ into X. The abstract equation of this form was studied by Kato [1] and [4]. According to Kato [1], $A(t, u)u$ is called the quasi-linear part of (E) and $f(t, u)$ the semi-linear part of (E). The operator f may be genuinely nonlinear, but is assumed to be regular and stable; hence f is regarded as a perturbation to the quasi-linear part. In this sense an equation of the form (E) is called a quasi-linear evolution equation. In [1], an existence theorem of strong solutions of (E) in a reflexive space is established by means of the method of successive approximation. This successive approximation is based on the theory of abstract linear "*hyperbolic*" equation advanced in [2] and [3].

The purpose of this paper is to introduce a notion of weak solution of (E) in a nonreflexive space and prove an existence and uniqueness theorem for the weak solutions of (E) by means of the method of Cauchy's polygonal approximation. This type of approximation makes it possible to prove the existence theorem under weaker conditions than those assumed in [1], [4] and [10]. In fact, it is assumed in [1], [4] and [10] that the operators $y\rightarrow f(t, y)$ and $y\rightarrow A(t, y)$ are X-Lipschitz continuous. But,

Received November 20, 1979
Revised August 31, 1980