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# A Proof of the Classical Kronecker Limit Formula

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## Introduction

Let $z, w$ be complex numbers. We assume that imaginary part of $z$ is positive. Set

$$\xi(s, w, z)=\sum_{m,n}{}' |m+nz+w|^{-2s} ,$$

where summation with respect to $m, n$ ranges over all pairs of integers such that $m+nz+w\neq 0$.

Put

$$\eta(z)=e[z/24]\prod_{n=1}^{\infty}(1-e[nz]) ,$$

$$\vartheta_1(w, z)=2e[z/12](\sin \pi w)\eta(z)\prod_{n=1}^{\infty}(1-e[w+nz])(1-e[-w+nz]) ,$$

where we write $e[z]=\exp(2\pi iz)$. Furthermore, we set $\xi'=d\xi/ds$. A version of the classical Kronecker limit formula is given as follows (see e.g., [9]).

If $w\notin \boldsymbol{Z}+\boldsymbol{Z}z$,

$$\xi'(0, w, z)=-\log\left|\frac{\vartheta_1(w, z)}{\eta(z)}\exp\frac{\pi iw(w-\bar{w})}{z-\bar{z}}\right|^2 .$$

If $w\in \boldsymbol{Z}+\boldsymbol{Z}z$,

$$\xi'(0, w, z)=-\log\{4\pi^2|\eta(z)|^4\} .$$

For the proofs of the Kronecker limit formula, we refer to [4] and papers quoted there. In this note we present a proof of the formula which makes use of the theory of the *double gamma function*. The author takes this opportunity to make an addendum of the reference to

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